TRINITY COLLEGE

Year 12 Mathematics Specialist 3,4
Test 3 2021

Section 1 Calculator Free

Vectors in 3D

STUDENT'S NAME _____

DATE: Wednesday 12 May **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Solve the system of equations

$$\begin{aligned} x + y + z &= 4 \\ 2x - y + z &= 0 \\ 3x + y + z &= 8 \end{aligned}$$

2. (8 marks)

Consider the following three planes:

$$\begin{aligned} 2x - y + 2z &= 4 \\ x + y - 2z &= 3 \\ x - 2y + kz &= m \end{aligned}$$

The system of equation has infinite solutions.

(a) Determine the values of k and m [3]

(b) Give a geometric interpretation of the solution for this system of equations for different values of m. [2]

(c) Determine the vector equation of the line where the three planes intersect. [3]

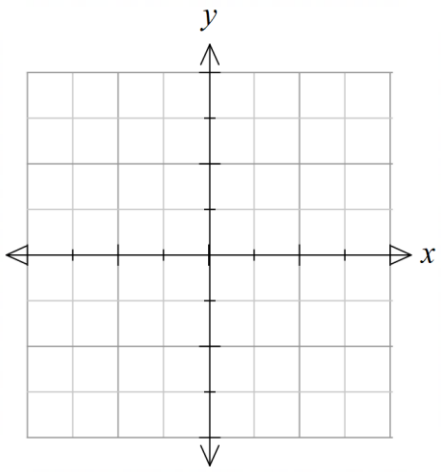
3. (9 marks)

Consider a particle whose position as a function of time is given by

$$\underset{\sim}{r}(t) = \begin{pmatrix} 2\sin(2t) \\ 2\cos(2t) \end{pmatrix}$$

(a) Prove the velocity of the particle is always tangential to its position vector. [3]

(b) Draw a sketch of the path of the particle and indicate the direction of motion. [3]



(c) Determine an expression for the total distance travelled by the particle between time a and time b . [3]

TRINITY COLLEGE

Year 12 Mathematics Specialist 3,4
Test 3 2021

Section 2 Calculator Assumed

Vectors in 3D

STUDENT'S NAME _____

DATE: Wednesday 12 May **TIME:** 30 minutes **MARKS**: 31

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (10 marks)

Two radio-controlled model planes take off at the same time from two different positions and with constant velocities. Model A leaves from the point with position vector $(-3\underset{\sim}{i} - 7\underset{\sim}{j})$ metres and has velocity $(5\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k})$ m/s; model B leaves from the point with position vector $(7\underset{\sim}{i} - \underset{\sim}{j} - 8\underset{\sim}{k})$ metres and has velocity $(3\underset{\sim}{i} - 4\underset{\sim}{j} + 6\underset{\sim}{k})$ m/s.

(a) Determine the distance between the two model planes after 1 second of flight. [3]

(b) Determine:

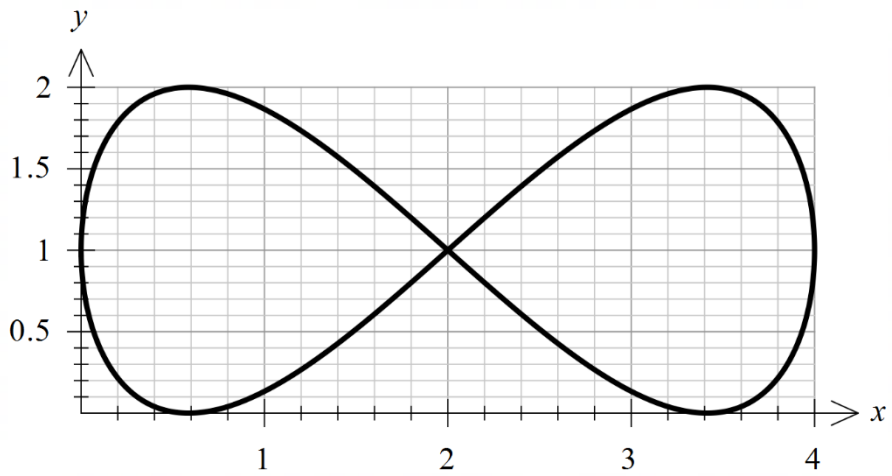
(i) an expression, in term of t, for the displacement between model plane A and model plane b. [1]

(ii) the shortest distance between the two model planes. [3]

(iii) the time when this occurs. [1]

5. (10 marks)

The path of a toy race car on a racetrack is shown below. The race car moves so that its position vector $\underset{\sim}{r}(t)$ is given by $\underset{\sim}{r}(t) = \begin{pmatrix} 2-2\cos(t) \\ 1-\sin(2t) \end{pmatrix}$ metres, where t is the number of seconds the particle has been in motion.



(a) Determine the starting position of the race car and mark this as point S in the diagram above. [1]

(b) Determine the initial velocity of the race car and indicate this on the diagram above. [3]

(c) Determine the Cartesian equation for the path of the race car. [3]

(d) Determine the distance the race car travels in completing one circuit of the racetrack. [3]

6. (11 marks)

A ball rolls off a table with a speed of 60 cm/s. The table is 1 m high. The ball undergoes acceleration due to gravity of $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} m/s^2$

(a) Determine the point at which the ball hits the floor and determine the speed at the instant. [5]

(b) Determine the angle θ between the path of the ball and a vertical line drawn through the point of impact. [3]

(c) Suppose the ball rebounds from the floor at the same angle with which it hits the floor but loses 20% of its speed due to energy absorbed by the ball on impact. Where does the ball strike the floor on the second bounce? [3]